**FEM problem set A**

Yue Jiao – 911024-7799

1. Heat equation

The following equation is going to be solved.

1. First, finding a weak formulation of the problem is necessary. The test space should satisfy the Dirichlet boundary condition.

The trial space shall be the following.

Let Let’s just assume linear relation for since it is easy to see this . Then the variational formulation shall satisfy the following equation for all .

So, the weak formulation is then to find such that ,

The solution to the differential equation is then where is the variable and is the Dirichlet boundary condition.

1. To compute this equation numerically with finite element method, let’s introduce a uniform grid where and . Then the following finite element method can be performed.

Let and . The function is the standard 1D hat function except at the last node . The function should has the form like:

Then the FEM will become:

Which is a linear equation in the form of where

And is the solution we need to create where is the variable.

1. Poisson equation

In this problem, we want to solve the Poisson equation:

with , and the rectangular domain with corners in the points

, , and , and with boundary .

1. To solve this equation numerically, an element mesh is first developed. So, an integer is chosen. Then for each element square, the block is divided into identical smaller squares. Since the area of the domain is , a number of small squares will be created. The last step is to connect the upper-left corner and the lower-right corner of each of these small squares. Then a mesh with triangular elements is created. The number of internal nodes is then . A figure is showed below. This mesh shall be called as .

II

III

I

3

2

1

VI

IV

V

1. A discrete finite element approximation space can be formulated as following.
2. So, the Galerkin FEM can be formulated as following.
3. Let where and for node . here is the number of the internal nodes which is in our case. Let’s choose to be

Which is precisely linear equation :

So, which means the following linear equation solves the PDE.

In our case, the function can be asserted to the following values:

So, . Since area of each triangle is . So we will have the following result:

Then, matrix can be calculated.

For the vector , similar method can be used. We will get:

Which can be done easily analytically or numerically.